Analysis of Runaway Electron Synchrotron Emission in Alcator C-Mod

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Runaway electrons in C-Mod

Alcator C-Mod plasma parameters:

- $B_{\text{tor}} = 2 - 8$ T
- $I_p = 0.5 - 2$ MA
- $\bar{n}_e = 0.2 - 4 \times 10^{20}$ m$^{-3}$
- $T_{e0} = 1 - 8$ keV
- $R = 0.68$ m, $a = 0.22$ m

Synchrotron radiation (SR) can be in the visible/near-infrared range (300-1000 nm).
Motivation

Q: From SR, can we distinguish a mono-energetic (and mono-pitch) RE distribution from a continuum distribution of energies and pitches?

• Recent studies [1-3] have predicted that REs will accelerate to a maximum energy at which the radiative force and collisional friction balances the electric force, forming a “bump” on the tail of the energy distribution function.

• Others [4,5] suggest that a broader distribution contributes to the SR spectra.

• Knowing the maximum energy of REs – as determined by the distribution – can have important implications for RE mitigation in fusion devices.

Motivation

Q: From SR, can we distinguish a \textit{mono-energetic} (and mono-pitch) RE distribution from a \textit{continuum distribution} of energies and pitches?

A: Not yet...
Experimental setup

Data is collected using an absolutely-calibrated spectrometer installed on C-Mod, with spectral range of \(~350\text{-}1020\) nm.
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View outside vessel  View inside vessel  Toroidal cross section of C-Mod
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Look at 3 different runaway shots
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- Flattop RE
- Ramp-up RE
- Early RE

Synchrotron Emission
Look at 3 different runaway shots

- Flattop RE
- Ramp-up RE
- Early RE

Synchrotron Emission
Flattop synchrotron emission data

Plasma parameters at $t = 1.5$ s:

- $B_t = 5.35$ T
- $I_p \approx 1$ MA (end of flat-top)
- $\bar{n}_e = 2.5 \cdot 10^{19}$ m$^{-3}$
- $T_{e0} = 4.25$ keV
- $a_{beam} \approx 5$ cm (as seen by camera)
- $V_{loop} = 1.05$ V
  \[ \Rightarrow E = 0.25 \text{ V/m} \]
  \[ \Rightarrow \frac{E}{E_c} = 12 \]

The red highlighted data is at $t = 1.5$ s and is used in this analysis.
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Two models for the RE distribution

For a **mono-energetic** and mono-pitch RE beam, the brightness (W/m$^3$/sr) is [6]:

$$B_{\text{mono}}(\lambda, \theta, p) = \frac{2 R n_r}{\pi \theta_{\text{eff}}(p, \theta)} P(\lambda, \theta_{\text{eff}}, p)$$

where $n_r$ is the density of REs emitting SR, $\theta = v_\perp/v_\parallel = p_\perp/p_\parallel$ is the pitch, and $p = \sqrt{E^2/m^2c^4} - 1$ is the normalized momentum.

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For a **distribution**, \(f_{RE}\), of energies and pitches [7],

\[
f_{RE}(p_\parallel, p_\perp) = \frac{n_r E}{2\pi c z p_\parallel ln\Lambda} \exp \left( - \frac{p_\parallel}{c z ln\Lambda} - \frac{E p_\perp^2}{2 p_\parallel} \right),
\]

the brightness \((W/m^3/sr)\) is [4]:

\[
B_{\text{dist}}(\lambda) = 4R \int \frac{1}{\theta_{\text{eff}}(p_\parallel, p_\perp)} P(\lambda, \theta_{\text{eff}}, p) f_{RE}(p_\parallel, p_\perp) dp_\parallel dp_\perp
\]

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The RE **density**, \( n_r \), is estimated [4,6] using the plasma current carried by the REs, \( I_r \), and cross-sectional area, \( A_r \), of the beam (as seen by our cameras):

\[ n_r = I_r / (ecA_r) \]

During the discharge, we do not know \( I_r \), so we have to fit the data by varying the RE current for both the mono-energetic and continuum distributions.

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Mono-energetic fit matches **flattop** data

- This data can be well-fit for a range of RE energies, pitches, and currents:
  
  \[24.8 \text{ MeV} \leq E_{\text{mono}} \leq 30.6 \text{ MeV}\]
  \[0.070 \leq \theta = \frac{v_\perp}{v_\parallel} \leq 0.125\]
  \[77 \text{ A} \leq I_{r,\text{ mono}} \leq 82 \text{ A}\]

- Assuming all REs emit SR at 28 MeV and pitch of 0.09, this means they only carry \(\sim 100 \text{ A}\) of the 1 MA plasma current.

\[
\begin{align*}
E_{\text{avg}} &= 28.0 \text{ MeV} \\
\theta_{\text{avg}} &= 0.09 \\
I_{r,\text{avg}} &= 81 \text{ A}
\end{align*}
\]
Continuum distribution matches flattop data

This best fit calculates:

- $E_{\text{max,dist}} = 19.7$ MeV
  - About 10 MeV less than $E_{\text{mono}}$

- $I_{r,\text{dist}} = 3.5$ kA
  - Accounts for <1% of the total plasma current, but more than $I_{r,\text{mono}}$

- $Z_{\text{eff,dist}} = 3$
  - Lower bound of fitting range (3 – 7).
Both **flattop** fits are comparable

The **mono-energetic** and **continuum distribution** fits are very similar, with about the same goodness of fit.
Both ME and CD fits are again comparable.

**Early Runaways**

- $E_{\text{max, dist}} = 14.6$ MeV
- $I_{r, \text{dist}} = 4.6$ kA
- $Z_{\text{eff}} \sim 10$

$E_{\text{avg}} = 28.1$ MeV
$\theta_{\text{avg}} = 0.1$
$I_{r, \text{avg}} = 118$ A

Plasma parameters at $t = 0.18$ s:

- $B_t = 5.24$ T
- $I_p \approx 670$ kA
- $\bar{n}_e = 5.9 \cdot 10^{19}$ m$^{-3}$
- $T_{e0} = 2.5$ keV
- $a_{\text{beam}} \approx 6$ cm (as seen by camera)
- $V_{\text{loop}} \approx 2.3$ V

$\Rightarrow E = 0.54$ V/m
$\Rightarrow E/E_c \approx 11$
Both ME and CD fits are again comparable.

Ramp-up Runaways

Plasma parameters at t = 0.54 s:

- $B_t = 5.36$ T
- $I_p \approx 800$ kA
- $n_e = 6.6 \times 10^{19}$ m$^{-3}$
- $T_{e0} = 2.3 - 3.2$ keV
- $a_{beam} \approx 7$ cm (as seen by camera)
- $V_{loop} \approx 1.1$ V
  \[ \Rightarrow E = 0.26 \text{ V/m} \]
  \[ \Rightarrow E/E_c \approx 4.8 \]
Use CODE [5,9] to solve the forward problem

- Time dependent parameters:
  - $T_{e0}(t)$
  - $\bar{n}_e(t)$
  - $V_{\text{loop,0}}(t) \rightarrow E(t)$
  - $Z_{\text{eff}}(t)$
  - $B \rightarrow \text{Synchrotron}$

- Secondary avalanching source:
  - Rosenbluth-Putvinskii (RP) [10]
  - Chiu-Harvey (CH) [11,12]

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Bump forms on tail of Early Runaway distribution

- Avalanche populates lower energies

\[ F \approx 2 E_{\text{MeV}} \]
Bump forms on tail of **Early Runaway** distribution

- Avalanche populates lower energies
- **RP Avalanche** extends tail

Parallel CODE distribution, \( t = 0.1789 \) s

\[
\frac{1}{E^{2}} \text{MeV}
\]

\( p \) vs. \( \sim 2E_{\text{MeV}} \)
Bump forms on tail of **Early Runaway** distribution

- Avalanche populates lower energies
- **RP Avalanche** extends tail
- **CH Avalanche** matches **No Avalanche** case at high energies
  - Primary (Dreicer [13]) generation dominates

![Parallel CODE distribution, t = 0.1789 s](image)

\[ F \sim 2E^{\frac{1}{2}}_{\text{MeV}} \]

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- **RP Avalanche** extends tail
- **CH Avalanche** matches **No Avalanche** case at high energies
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- **No Synchrotron** case still forms bump
  → Dynamic plasma parameters can form bump

\[ \sim 2E_{\text{MeV}} \]

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  - Primary (Dreicer [13]) generation dominates
- **No Synchrotron** case still forms bump
  - Dynamic plasma parameters can form bump
  - **Synchrotron limits bump energy**

Synchrotron power loss more easily seen in 2D

- No Avalanche, $t = 0.0600 \text{ s}$
- Rosenbluth-Putvinskii, $t = 0.0600 \text{ s}$
- CH, No Synchrotron, $t = 0.0600 \text{ s}$
- Chiu-Harvey, $t = 0.0600 \text{ s}$
Synchrotron power loss more easily seen in 2D
Summary and future work

Mono-energetic and continuum distribution calculations both fit C-Mod experimental data equally well. A time-dependent CODE model of one C-Mod runaway discharge calculates a bump on the tail of the energy distribution function which is:

- Dominated by primary generation
- Limited by synchrotron radiation
- Formed by dynamic plasma parameters

Next steps:

- Calculate the synchrotron brightness from CODE’s distribution functions and compare to experiment
- Run CODE for the other runaway discharges
- Use a non-linear solver for discharges with runaway fractions > 10-15% (see Adam Stahl’s presentation Wednesday, 9:30am)
References

Backup slides
Runaway electrons in C-Mod

Alcator C-Mod plasma parameters:

\[ \text{B}_{\text{tor}} = 2 - 8 \text{ T} \]
\[ I_p = 0.5 - 2 \text{ MA} \]
\[ \bar{n}_e = 0.2 - 2 \cdot 10^{20} \text{ m}^{-3} \]
\[ T_{e0} = 1 - 5 \text{ keV} \]

Synchrotron radiation (SR) can be in the visible/near-infrared range (300-1000 nm).
Look at 3 different runaway shots
Ramp-up synchrotron emission data

Plasma parameters at $t = 0.54$ s:

- $B_t = 5.36$ T
- $I_p \approx 800$ kA
- $\bar{n}_e = 6.6 \cdot 10^{19}$ m$^{-3}$
- $T_{e0} = 2.3 - 3.2$ keV
- $a_{\text{beam}} \approx 7$ cm (as seen by camera)
- $V_{\text{loop}} \approx 1.1$ V

$\Rightarrow E = 0.26$ V/m
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Early synchrotron emission data

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Both **flattop** fits are comparable

- The **mono-energetic** and **continuum distribution** fits are very similar, with about the same goodness of fit.

- There is a **brightness feature** that cannot be fit by either.
  - Maybe we need a different RE distribution?
  - Or perhaps this is a result of a calibration error?
Both **flattop** fits are comparable

- \[ \text{resnorm} = \sum_{\lambda} [\text{data}(\lambda) - \text{fit}(\lambda)]^2 \]
  - Goodness of fit
  - MATLAB’s \textit{lsqcurvefit} was used to perform a nonlinear least squares fit to the two models.
  - We assume that each data point has the same uncertainty.

- \[ \text{resnorm}_{\text{mono}} = 1.4 \cdot 10^{-12} \]
- \[ \text{resnorm}_{\text{dist}} = 1.2 \cdot 10^{-12} \]
Both ramp-up fits are comparable

- The mono-energetic and continuum distribution fits are again very similar.
  - $E_{\text{avg}} = 30.2$ MeV
  - $\theta_{\text{avg}} = 0.1$
  - $I_{r,\text{avg}} = 40$ A
  - $E_{\text{max, dist}} = 12.6$ MeV
  - $I_{r,\text{dist}} = 933$ A
  - $T_{e,\text{dist}} = 3.2$ keV
Both ME and CD fits are again comparable.

**Early Runaways**
- $E_{\text{avg},\text{dist}} = 28.1$ MeV
- $\theta_{\text{avg}} = 0.1$
- $I_{r,\text{avg}} = 118$ A
- $E_{\text{max},\text{dist}} = 14.6$ MeV
- $I_{r,\text{dist}} = 4.6$ kA
- $Z_{\text{eff}} \sim 10$

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- $I_{r,\text{dist}} = 933$ A
- $T_{e,\text{dist}} = 3.2$ keV
“Bump on tail” formation

• In [1], the energy at which REs converge is calculated as a function of $Z_{\text{eff}}$, which we were not able to measure for the flattop data (shot 1151002022).

• For the plasma parameters at $t = 1.5$ s, a mono-energetic RE beam of 28 MeV is produced by a $Z_{\text{eff,mono}}$ of $\sim 4$, which is consistent with experiments on C-Mod. [8]

• This also means that C-Mod’s high $Z_{\text{eff}}$ (3-7) in RE-producing plasma conditions could limit the RE energy to $< 30$ MeV.

Runaway electrons

- In plasmas, the Coulomb collision frequency between particles varies as \( \frac{\text{density}}{\text{velocity}^3} \).
- This can lead to a cascade of relativistic “runaway” electrons (REs) with energies of tens of MeV.
- Relativistic charged particles emit a cone of synchrotron radiation (SR) in their direction of motion.
- In C-Mod, this radiation can be in the visible/near-infrared range (300-900 nm).
Camera view of SR

Synchrotron emission
Camera view of SR

Parameters:
- $R = 68 \text{ cm} - \text{C-Mod major radius}$
- $a = 22 \text{ cm} - \text{C-Mod minor radius}$
- $a_{\text{beam}} \approx 5 \text{ cm} - \text{radius of RE beam}$
- $A_{\text{beam}} \approx 80 \text{ cm}^2 - \text{area of RE beam}$
- $r_{\text{lens}} = 9 \text{ mm} - \text{lens aperture}$
- $r_0 = 1.77 \text{ m} - \text{distance from lens to tangency radius (SR)}$
Time evolution: Mono-energetic RE beam

- $t = 1.33\text{s}$
  - $E_{\text{avg}} = 24.3\text{ MeV}$, $\sigma_E = 1.8\text{ MeV}$
  - $\theta_{\text{avg}} = 0.08$, $\sigma_\theta = 0.02$
  - $I_{\text{avg}} = 48\text{ A}$, $\sigma_f = 2\text{ A}$
Time evolution: Mono-energetic RE beam

- $t = 1.33s$
  - $E_{avg} = 24.3$ MeV, $\sigma_E = 1.8$ MeV
  - $\theta_{avg} = 0.08$, $\sigma_\theta = 0.02$
  - $I_{avg} = 48$ A, $\sigma_I = 2$ A

- $t = 1.50s$
  - $E_{avg} = 28.0$ MeV, $\sigma_E = 1.2$ MeV
  - $\theta_{avg} = 0.09$, $\sigma_\theta = 0.01$
  - $f_{avg} = 81$ A, $\sigma_f = 1$ A
Non-monotonic brightness?
Full power calculation

The power radiated by a relativistic electron in a tokamak is given by [A]:

\[ P_{\text{full}}(\lambda) = \frac{ce^2}{\epsilon_0\lambda^3\gamma^2} \left\{ \int_0^\infty \frac{1 + 2y^2}{y} J_0(ay^3) \sin \left( \frac{3}{2} \xi \left( y + \frac{1}{3}y^3 \right) \right) dy \right. \\
+ \left. \frac{4\eta}{1 + \eta^2} \int_0^\infty y J_1(ay^3) \cos \left( \frac{3}{2} \xi \left( y + \frac{1}{3}y^3 \right) \right) dy \right\} - \frac{\pi}{2} \}

where

\[ a = \frac{\xi\eta}{(1 + \eta^2)}, \quad \xi = \frac{4\pi}{3} \frac{R}{\lambda\gamma^3\sqrt{1+\eta^2}}, \quad \eta \approx \frac{eB}{m\gamma c} \frac{v_\perp}{v_\parallel} \]

and \( \frac{v_\perp}{v_\parallel} \) is the pitch and \( \gamma = E/mc^2 \) is the relativistic Lorentz factor.

Power density approximation

• Using the approximation:

\[ \lambda \ll \frac{4\pi}{3} R \eta/[\gamma^3 (1 + \eta)^3] \]

the power calculation reduces to [B] :

\[ P_{as2} (\lambda) = \frac{\sqrt{3}}{8\pi \varepsilon_0 \lambda^2 R} \frac{c e^2 \gamma (1+\eta)^2}{\sqrt{\eta}} \exp\left( - \frac{4\pi}{3} \frac{R}{\lambda \gamma^3} \frac{1}{1+\eta} \right) \]

• This approximation is only valid for C-Mod at low energies (~25MeV).

Mono-energetic brightness

For a mono-energetic (mono-pitch) beam, the brightness (W/m³/sr) is [C]:

\[ B(\lambda, \theta_{eff}, \gamma) = \frac{2 R n_r}{\pi \theta_{eff}} P(\lambda, \theta_{eff}, \gamma) \]

where

\[ \theta_{eff} \approx \sqrt{\left(\frac{v_{\perp}}{v_{\parallel}}\right)^2 + \gamma^{-2} + \left(\frac{n_{\text{ens}}}{r_0}\right)^2} \]

is the effective viewing aperture and \( n_r \) is the runaway beam density at this energy.

Distribution of energies and pitches

For a distribution of energies and pitch angles [D]:

\[ f_{RE}(p_\parallel, p_\perp) = \frac{n_r \hat{E}}{2\pi c_z p_\parallel \ln \Lambda} \exp \left( - \frac{p_\parallel}{c_z \ln \Lambda} - \frac{\hat{E} p_\perp^2}{2p_\parallel} \right) \]

The brightness is calculated [E]:

\[ B(\lambda) = 4R \int_0^1 \int_{p_{\text{min}}}^{p_{\text{max}}} \frac{1}{\theta_{eff}(\chi)} P(\lambda, \theta_{eff}(\chi), \gamma(p)) f_{RE}(p, \chi)p^2 dp d\chi \]

Acknowledgments

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Dr. Bob Mumgaard is gratefully acknowledged for his assistance in absolutely calibrating the Ocean Optics spectrometers and calculating the brightness.